• The examination lasts for 5 hours. There are 2 problems worth 20 points in total. There are two tables at your disposal (in two neighbouring cubicles); the apparatus for Problem E1 is on one table and the apparatus for Problem E2 is on the other table; you may move freely between these tables. However, you are not allowed to move any piece of experimental equipment from one table to the other.

• Initially the experimental equipment on one table is covered and that on the other table is boxed. You must not remove the cover, open the box or open the envelope containing the problems before the sound signalling the beginning of the competition (three short signals).

• You are not allowed to leave your working place without permission. If you need any assistance (malfunctioning equipment, broken calculator, need to visit a restroom, etc), please raise the corresponding flag (“help” or “toilet”) with a long handle at your seat above your cubicle walls and keep it raised until an organizer arrives.

• Use only the front side of the sheets of paper.

• For each problem, there are dedicated Solution Sheets (see header for the number and pictogramme). Write your solutions onto the appropriate Solution Sheets. For each Problem, the Solution Sheets are numbered; use the sheets according to the enumeration. Always mark on which Problem Part and Question you are working. Copy the final answers into the appropriate boxes of the Answer Sheets. There are also Draft papers; use these for writing things which you don’t want to be graded. If you have written something that you don’t want to be graded on the Solution Sheets (such as an initial or incorrect solution), cross it out.

• If you need more paper for a certain problem, please raise the “help” flag and tell an organizer the problem number; you will be given two Solution sheets (you may do this more than once).

• You should use as little text as possible: try to explain your solution mainly with equations, numbers, symbols and diagrams.

• Avoid unnecessary movements during the experimental examination and do not shake the cubicle walls: the laser experiment requires stability.

• Do not look into the laser beam or its reflections! It may permanently damage your eyes!

• The (first) single sound signal tells you that there are 30 min of solving time left; the (second) double sound signal means that 5 min is left; the (third) triple sound signal marks the end of solving time. After the third sound signal you must stop writing immediately. Put all the papers into the envelope at your desk. You are not allowed to take any sheet of paper out of the room. If you have finished solving before the final sound signal, please raise your flag.
Problem E1. The magnetic permeability of water (10 points)

The effect of a magnetic field on most substances except ferromagnets is rather weak because the energy density of the magnetic field in a substance of relative magnetic permeability \( \mu \) is given by the formula \( w = \frac{1}{2 \mu_0} B^2 \), and \( \mu \) is typically very close to 1. Nevertheless, with suitable experimental techniques such effects are clearly observable. In this problem we study the effect of a magnetic field, created by a permanent neodymium magnet, on water and use the results to calculate the magnetic permeability of water. **You are not required to estimate any uncertainties throughout this problem and you may neglect the effects of surface tension.**

**The setup** comprises 1 a stand (the highlighted numbers correspond to the numbers in the figure), 3 digital calipers, 4 a laser pointer, 5 a dish with water and 7 a cylindrical permanent magnet in it (the magnet is axially magnetised). The dish is fixed to the base of the stand by the magnet’s pull. The laser is fixed to the caliper, the base of which is fastened to the stand; the caliper allows horizontal displacement of the laser. The on-off button of the laser can be kept down using 13 the white conical tube. The depth of the water above the magnet should be reasonably close to 1 mm (if shallower, the water surface becomes so curved that it will be difficult to take readings from the screen). 15 A cup of water and 16 a syringe can be used to adjust the water level (to raise the level by 1 mm, add 13 ml of water). 2 A sheet of graph paper (the “screen”) is to be fixed to the vertical plate with 14 small magnetic tablets. If the laser spot on the screen is smeared, check for dust on the water surface (and blow it away).

The remaining legend for the figure is as follows: 6 the point where the laser beam hits the screen; 11 the LCD screen of the caliper; 10 button which switches the caliper units between millimeters and inches; 8 on-off switch; 9 button for setting the origin of the caliper reading. Beneath the laser pointer, there is one more button on the caliper, which temporarily re-sets the origin (if you have pushed it inadvertently, push it once again to return to the normal measuring mode).

**Numerical values** for your calculations: **horizontal distance** between the magnet’s centre and the screen \( L_0 = 490 \text{ mm} \). Check (and adjust, if needed) the alignment of the centre of the magnet in two perpendicular directions. The vertical axis of the magnet must intersect with the laser beam, and it must also intersect with 12 the black line on the support plate. Magnetic field at the magnet’s axis, at a height of 1 mm from the flat surface \( B_0 = 0.50 \text{ T} \); **Density of water** \( \rho_w = 1000 \text{ kg/m}^3 \); **gravitational acceleration** \( g = 9.8 \text{ m/s}^2 \); **vacuum permeability** \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \).

**WARNINGS:**
- The laser orientation is pre-adjusted, do not move it!
- DO NOT look into the laser beam or its reflections!
- Do not try to remove the strong neodymium magnet!
- Do not put magnetic materials close to the magnet!
- Turn off the laser when not used, batteries drain in 1 hr!

**Part A. Qualitative shape of the water surface (1 points)**

When a cylindrical magnet is placed below the water surface, the surface becomes curved. Observe the shape of the water surface above the magnet and hence decide whether the water is diamagnetic (\( \mu < 1 \)) or paramagnetic (\( \mu > 1 \)).

**Part B. Exact shape of the water surface (7 points)**

The curvature of the water surface can be observed with high sensitivity by measuring the reflection of the laser beam by the surface. We use this effect to calculate the dependence of the height of the water above the magnet on horizontal position.

i. (1.6 pts) Measure the dependence of the height \( y \) of the laser spot on the screen on the caliper reading \( x \) (see figure). You should cover the entire usable range of caliper displacements. Record the results in the table on the Answer Sheet.

ii. (0.7 pts) Plot the graph of the obtained dependence.

iii. (0.7 pts) Using your graph, determine the angle \( \alpha_0 \) between the beam and the horizontal plane.

iv. (1.4 pts) Please note that the slope of the water surface (\( \tan \beta \)) can be expressed as follows:

\[
\tan \beta \approx \beta \approx \frac{\cos^2 \alpha_0}{2} \frac{y - y_0 - (x - x_0) \tan \alpha_0}{L_0 + x - x_0},
\]

where \( y_0 \) is the height of the laser spot on the screen when the beam is reflected from the water surface at the axis of the magnet, and \( x_0 \) is the corresponding position of the caliper.

Calculate the values of the slope of the water surface and record them in the Table on the Answer Sheet. Please note that it may be possible to simplify your calculations if you substitute a reading from the last graph for some combination of terms in the expression above.

v. (1.6 pts) Calculate the height of the water surface relative to the water surface far from the magnet, as a function of \( x \) and record it in the Table on the Answer Sheet.

vi. (1 pt) Plot the graph of this dependence. Indicate on it the place where the beam hits the water surface directly above the magnet.

**Part C. Magnetic permeability (2 points)**

Using the results of Part B, calculate the value of \( \mu - 1 \) (the magnetic susceptibility), where \( \mu \) is the relative magnetic permeability of the water. Write your final formula and the numerical result into the Answer Sheet.
Problem E2. Nonlinear Black Box (10 points)

In simple problems, electrical circuits are assumed to consist of linear elements, for which electrical characteristics are directly proportional to each other. Examples include resistance ($V = RI$), capacitance ($Q = CV$) and inductance ($V = LI$), where $R$, $C$ and $L$ are constants. In this problem, however, we examine a circuit containing nonlinear elements, enclosed in a black box, where the assumption of linear proportionality no longer holds.

The setup comprises a multimeter (labelled “IPhO-measure”), a current source, a black box containing nonlinear elements, and four test leads with stackable connectors for wiring. Be careful not to break the seal on the black box.

The multimeter can measure current and voltage simultaneously. Using it you can record up to 2000 data points, each consisting of: voltage $V$, current $I$, power $P = IV$, resistance $R = V/I$, voltage time-derivative $\dot{V}$, current time-derivative $\dot{I}$ and time $t$. See the manual for further details. If you exceed 2000 saved data points, the oldest data will be overwritten.

The constant current source supplies stable current as long as the voltage across its terminals stays between $-0.6125\text{V}$ and $0.6125\text{V}$. When switched off, the constant current source behaves as a large (essentially infinite) resistance.

The black box contains an electric double layer capacitor (which is a slightly nonlinear high capacitance capacitor), an unknown nonlinear element, an inductor $L = 10\mu\text{H}$ of negligible resistance and a switch as indicated on the circuit diagram. The nonlinear element can be considered to be a resistor with a nonlinear dependence between the voltage and the current ($I$ is a continuous function of $V$ with $I(0) = 0$). Likewise, for the capacitor, the differential capacitance $C(V) = dQ/dV$ is not exactly constant. **We say that the voltage on the black box is positive when the potential on its red terminal is higher than the potential on the black terminal. Positive voltage will be obtained when the terminals of matching colours on the black box and the current source are connected (you are allowed to use negative voltages).**

Here it is safe to discharge the capacitor in the black box by shorting its inputs either directly or through the IN and OUT terminals on the multimeter: the internal resistance of this capacitor is enough to keep the current from damaging anything.

You are not required to estimate any uncertainties throughout this problem.

**Part A. Circuit without inductance (7 points)**

In this part, keep the switch on the black box closed (push “I” down), so that the inductance is bypassed.

Please note that some measurements may take considerable time, therefore it is recommended that you read through all the tasks in part A to avoid unnecessary work.

i. (1 pt) Confirm that the output current of the current source is approximately $6\text{mA}$, and determine the range within which it varies, for voltages between $0$ and $+480\text{mV}$. Draw the circuit diagram.

ii. (1.2 pts) Show that the differential capacitance $C(V)$ used in the black box is approximately $2\text{F}$ by measuring its value for a single voltage of your choice $C(V_0) = C_0$. Draw the circuit diagram.

iii. (2.2 pts) Neglecting the nonlinearity of the capacitance (i.e. $C(V) \approx C_0$), determine the current–voltage characteristic of the nonlinear element used in the black box. Plot the $I(V)$ curve for obtainable positive voltages across the black box on the answer sheet. Draw the circuit diagram.

iv. (2.6 pts) Using measurements taken over the whole range of obtainable voltages, calculate and plot the $C(V)$ curve for obtainable positive voltages across the black box on the answer sheet. Write down the minimal and maximal values of the differential capacitance, $C_{\text{min}}$ and $C_{\text{max}}$. Draw the circuit diagram.

**Part B. Circuit with inductance (3 points)**

Introduce the inductor by opening the switch on the black box (push “0” down). Using the same method as in part A-iii, measure and plot the current–voltage characteristic of the nonlinear element. Describe any significant differences between the curves of parts A and B and suggest a reason using qualitative arguments.

Here you need to know that the nonlinear element also has a capacitance of about $1\text{nF}$ which is connected in parallel to the nonlinear resistance.
**IPhO-measure: short manual**

*IPhO-measure* is a multimeter capable of measuring voltage $V$ and current $I$ simultaneously. It also records their time-derivatives $\dot{V}$ and $\dot{I}$, their product $P = VI$, ratio $R = V/I$, and time $t$ of the sample. Stored measurements are organized into separate sets; every stored sample is numbered by the set number $s$ and a counter $n$ inside the set. All saved samples are written to internal flash memory and can be retrieved later.

**Electrical behaviour**

The device behaves as an ammeter and a voltmeter connected as follows.

<table>
<thead>
<tr>
<th>Range</th>
<th>Internal resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltmeter 0 — 2 V</td>
<td>1 MΩ</td>
</tr>
<tr>
<td>Voltmeter 2 — 10 V</td>
<td>57 kΩ</td>
</tr>
<tr>
<td>Ammeter 0 — 1 A</td>
<td>1 Ω</td>
</tr>
</tbody>
</table>

**Basic usage**

- Tap “POWER” to switch the *IPhO-measure* on. The device is not yet measuring; to start measuring, tap “START”. Alternatively, you can now start browsing your stored data, see below.
- To browse previously saved samples (through all sets), press “PREVIOUS” or “NEXT”. Hold your finger down to jump directly between sets.
- While not measuring, tap “START” to start measuring a new set.
- While measuring, tap “SAMPLE” to store a data sample (with the current readings).
- While measuring, you can also browse other samples of the current set, using “PREVIOUS” and “NEXT”.
- Tap “STOP” to end a set and stop measuring. The device is still on, you are ready to start a new measuring session, or browse stored data.
- Tapping “POWER” turns the device off. The device will display “my mind is going . . .”; don’t worry, all the data measurements will still be stored and you will be able to browse them after you switch the device on again. Saved samples will not be erased.

**Display**

A displayed sample consists of nine variables:

1. index $n$ of the sample in the set;
2. index $s$ of the set;
3. time $t$ since starting the set;
4. voltmeter output $V$;
5. rate of change of $V$ (the time-derivative $\dot{V}$); if derivative cannot be reliably taken due to fluctuations, “+nan/s” is shown;
6. ammeter output $I$;
7. rate of change of $I$ (the time-derivative $\dot{I}$); if derivative cannot be reliably taken due to fluctuations, “+nan/s” is shown;
8. product $P = VI$;
9. ratio $R = V/I$.

If any of the variables is out of its allowed range, its display shows “+inf” or “-inf”.

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