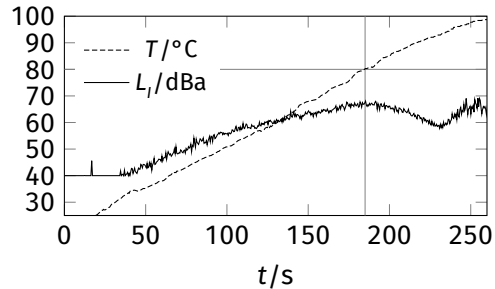


**12-1° Kettle** (3 p) While water is being heated up, one can notice that the noise level  $L_1$  produced by the kettle increases until the temperature reaches ca. 80 °C and then decreases again (see the plot). Explain why it happens.

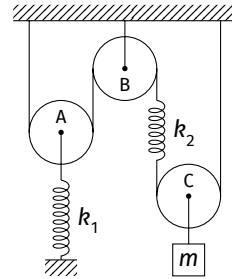


**12-2° Falling target** (3 p) Jānis is shooting arrows at target located at height  $H$  above the ground and at horizontal distance  $L$  from Jānis. The arrow is to be shot with initial velocity  $v$  from height  $h$  the moment when the target starts to fall. Free fall acceleration is  $g$ .

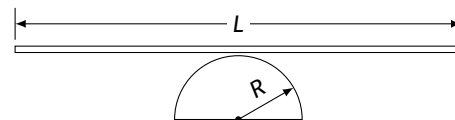
- At what angle should an arrow be shot in order to hit the target?
- In reality, reaction time  $\tau$  has to be taken into account. At what angle should Jānis shoot in this case?

**12-3° Pulleys and springs** (3 p) Three pulleys, two springs of stiffness  $k_1 = 15 \frac{\text{N}}{\text{m}}$  and  $k_2 = 10 \frac{\text{N}}{\text{m}}$ , respectively, and a block of mass  $m = 100 \text{ g}$  are connected with strings as shown in the figure. Masses of pulleys, springs and strings are negligibly small, strings are not stretchable. The system is in equilibrium.

- Determine the absolute deformation of each spring.
- Determine the distance the pulley C would rise, if the mass  $m$  is gradually decreased to zero.



**12-4° Rocking rod** (4 p) A thin uniform rod of length  $L$  is placed symmetrically on a half-cylinder of radius  $R$ . Determine the period of small oscillations of the rod. The central moment of inertia of a rod is  $\frac{1}{12}mL^2$ . Free fall acceleration is  $g$ . Assume the rod is not slipping.



**12-5° I-U source** (2 p) Power supply unit automatically switches between two regimes: (a) if the current in the outer circuit does not exceed  $I_{\text{max}}$ , it ensures constant potential difference  $U_0$ ; (b) if the potential difference across the outer circuit does not exceed  $U_{\text{max}}$ , it ensures constant current  $I_0$ . Determine the long-time average power dissipated on the load of resistance  $R$  if the switching from regime (a) to (b) and from (b) to (a) takes equal short time.

**12-6° Mercury merge** (4 p) What is the maximum radius of two identical drops of mercury on a glass surface that will spontaneously merge when brought in contact? Density and surface tension of mercury are  $\rho = 13,6 \frac{\text{g}}{\text{cm}^3}$  and  $\sigma = 0,5 \text{ J/m}^2$ , respectively; acceleration due to gravity  $g = 9,8 \frac{\text{m}}{\text{s}^2}$ . Assume that drops are spherical and that mercury does not wet glass.

**12-7° Candle light** (3 p) A thin convex lens with optical power  $D = 2 \text{ m}^{-1}$  and radius  $r = 15 \text{ cm}$  is located at distance  $a = 75 \text{ cm}$  from a candle. A screen is located at distance  $b = 200 \text{ cm}$  behind the lens. The lens is then removed. Determine the ratio of the maximum illuminance of the screen after and before the lens is removed. The principal optical axis of the lens passes through the candle flame and is perpendicular to the screen.

**12-8° Across the Universe** (4 p) In this problem, we assume that the Universe does not expand and is transparent for radiation. This model is rather simplistic, but is still useful for rough estimates. Assume that the average luminosity of stars in the Universe is  $L$ , and the number of stars per unit volume  $n$  is constant throughout the whole Universe. The energy density of radiation at any point of the universe is  $e$ .

- Using the given quantities, determine the radius  $R$  of a visible part of the Universe.
- Explain why  $R$  must be finite within this model.

**12-9° Origami (4 p)** Two infinite paper sheets with uniform surface charge density  $+\sigma$  are folded at an angle  $2\alpha$  and put together as shown in Fig. 1.

- (a) Determine the angle  $\alpha$  that would maximise the electric field intensity  $E$  at point A.

A  $2a \times a$  rectangle is cut from one of the sheets and folded at a  $90^\circ$  angle as shown in Fig. 2, forming two adjacent faces of a cube.

- (b) Determine the electric field intensity at the centre C of such an imaginary cube.

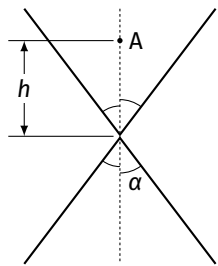


Fig. 1

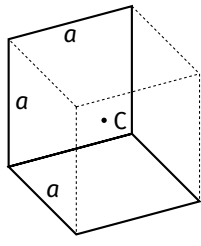


Fig. 2

**12-10° Multiple choice questions (3 p)** For each question there is one correct answer. Justify your choice. Answers without justification will receive zero marks.

**(1)** Given three resistors with resistances  $R_1 = 10\ \Omega$ ,  $R_2 = 50\ \Omega$  and  $R_3 = 100\ \Omega$ , how should you arrange them for the equivalent resistance to be between  $40\ \Omega$  and  $50\ \Omega$ ?

- (a)  $R_1$  and  $R_3$  in parallel, connected to  $R_2$  in series  
 (b)  $R_2$  and  $R_3$  in parallel, connected to  $R_1$  in series  
 (c)  $R_1$  and  $R_3$  in series, connected to  $R_2$  in parallel  
 (d)  $R_2$  and  $R_3$  in series, connected to  $R_1$  in parallel

**(2)** An incompressible fluid flows into a pipe of diameter 2 m with the speed  $5\ \frac{\text{m}}{\text{s}}$ . If the other end of the pipe has a cross-section area of  $0,5\ \text{m}^2$ , what is the speed of the fluid as it exits the pipe?

- (a)  $1,3\ \frac{\text{m}}{\text{s}}$   
 (b)  $12\ \frac{\text{m}}{\text{s}}$   
 (c)  $31\ \frac{\text{m}}{\text{s}}$   
 (d)  $40\ \frac{\text{m}}{\text{s}}$

**(3)** Healthy human eye (at a certain age) can form a sharp image of an object not closer than 25 cm away from it. Suppose a person cannot see objects clearly if they are closer than 50 cm away from the eye. A lens of what minimum optical power will allow the person to see clearly again at the distance of 25 cm?

- (a)  $-4\ \text{dpt}$   
 (b)  $-2\ \text{dpt}$   
 (c)  $+2\ \text{dpt}$   
 (d)  $+4\ \text{dpt}$

**(4)** A cylinder contains gas at pressure  $p$  and volume  $V$ . The gas undergoes isothermal expansion to volume  $3V$  followed by isobaric contraction back to its original volume, followed by isochoric increase in pressure until it has returned to its original pressure. Which of the following is true regarding the work  $A$  done by the gas?

- (a)  $A < \frac{2}{3}pV$   
 (b)  $\frac{2}{3}pV < A < \frac{4}{3}pV$   
 (c)  $\frac{4}{3}pV < A < 2pV$   
 (d)  $A > 2pV$

**(5)** Two metallic spheres X and Y are initially in contact and carry no charge. A negatively charged rod is brought near sphere X. If the two spheres are then separated, which of the following is true about the charges on each sphere?

- (a)  $q_X = 0, q_Y = 0$   
 (b)  $q_X > 0, q_Y > 0$   
 (c)  $q_X < 0, q_Y > 0$   
 (d)  $q_X > 0, q_Y < 0$

**(6)** A rectangular ( $a \times b$ ) wireframe is rotating around one of its sides in a uniform magnetic field of flux density  $\vec{B}$ . Is electromotive force (EMF) induced in the frame?

- (a) EMF is not induced  
 (b) EMF is induced  
 (c) Depends on the direction of  $\vec{B}$   
 (d) Depends on the  $\frac{a}{b}$  ratio  
 (e) Depends on the material of the wire