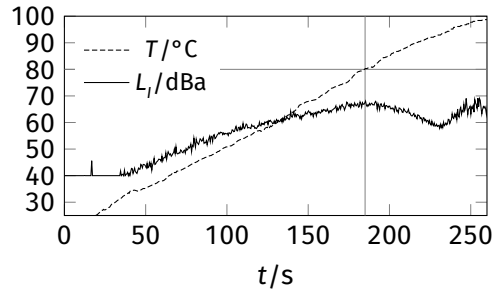


**10-1° Kettle** (3 p) While water is being heated up, one can notice that the noise level  $L_1$  produced by the kettle increases until the temperature reaches ca. 80 °C and then decreases again (see the plot). Explain why it happens.



**10-2° Weak heater** (3 p) A pot is filled with water at temperature  $T_0 = 20^\circ\text{C}$ , which is to be heated up with an immersive heater (metal spiral), the resistance  $R$  of which changes with temperature  $T$  so that  $R = R_0[1 + \alpha(T - T_0)]$ , where  $\alpha = 0,1 \frac{1}{^\circ\text{C}}$  and  $R_0 = 100 \Omega$ . Assume that the heater and water are in thermal equilibrium at all times. The pot is not insulated and loses energy  $Q = \beta(T - T_0)$ , where  $\beta = 2 \frac{1}{^\circ\text{C}}$  every second. The heater is connected to a current source, which outputs constant current  $I = 0,2 \text{ A}$ . Determine the maximum temperature  $T_{\text{max}}$  of water that can be achieved.

**10-3° Falling target** (3 p) Jānis is shooting arrows at target located at height  $H$  above the ground and at horizontal distance  $L$  from Jānis. The arrow is to be shot with initial velocity  $v$  from height  $h$  the moment when the target starts to fall. Free fall acceleration is  $g$ .

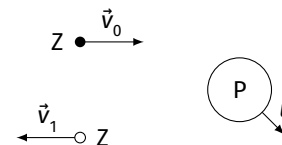
- At what angle should an arrow be shot in order to hit the target?
- In reality, reaction time  $\tau$  has to be taken into account. At what angle should Jānis shoot in this case?

**10-4° Motorboat** (5 p) A motorboat of mass  $m = 100 \text{ kg}$  is moving on a lake with velocity  $v_0 = 2 \frac{\text{m}}{\text{s}}$ . A drag force on the boat is proportional to the velocity:  $\vec{F} = -k\vec{v}$ , where  $k = 5 \frac{\text{kg}}{\text{s}}$ . The engine of the boat is stopped at  $t = 0$ , and the boat continues to move on a straight line.

- Determine the acceleration (both magnitude and direction) of the boat at  $t = 0$ .
- Determine the distance travelled by the boat from  $t = 0$  to the moment when the velocity of the boat was  $\frac{1}{2}v_0$ .
- Determine the velocity of the boat when it has travelled one third of its maximum displacement since  $t = 0$ .

**10-5° Global warming** (3 p) Two identical cylindrical glasses of cross-section area  $S = 30 \text{ cm}^2$  contain identical cubes of ice of mass  $m = 10 \text{ g}$  each. Both glasses are filled to half-height: the first one with distilled water (density  $\rho_0 = 1,00 \frac{\text{g}}{\text{cm}^3}$ ), the second one with salt water (density  $\rho_1 = 1,02 \frac{\text{g}}{\text{cm}^3}$ ) so that ice cubes do not touch the bottom of the glass. After some time ice melted in both glasses. Explain how the water level will change and determine the water level difference between the glasses when ice will have melted.

**10-6° Gravitational manoeuvre** (4 p) A probe  $Z$  is approaching the planet  $P$  from far away with velocity  $v_0 = 7 \frac{\text{km}}{\text{s}}$ . The probe passed near the planet, and when it was once again far away, the probe was moving in the opposite direction (see fig.). Assume the orbital velocity of the planet is  $u = 10 \frac{\text{km}}{\text{s}}$  and does not change either in magnitude or direction during the manoeuvre. Determine the maximal and minimal possible magnitude of the velocity  $v_1$  of the probe after the manoeuvre. All velocities and directions are relative to the star around which the planet is orbiting.



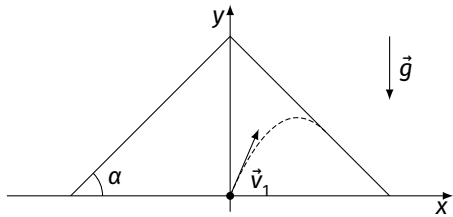
**10-7° Heavy pigeon** (4 p) A pigeon is perching at the middle of a stretchable string, the ends of which are fixed at the same height. Before the pigeon has perched on it, the string was not deformed, its length was  $l_0 = 5 \text{ m}$  and cross-section area  $S_0 = 2 \text{ mm}^2$ . The string snaps when the stress and the strain in it exceed  $\sigma = 2,5 \text{ MPa}$  and  $\epsilon = 0,5$ , respectively. The volume of the string does not change. Determine the maximum mass of a pigeon that can still perch on the string without making it snap. Free fall acceleration  $g = 9,8 \frac{\text{m}}{\text{s}^2}$ .

**10-8° Ball in a tank** (3 p) Consider a cylindrical tank with water, which can rotate around its axis. At the distance  $R$  from the axis, a light thread of length  $l$  is attached to the bottom of the tank. At the other end of the thread, a small ball of density that is less than the density of water is attached. The ball is always completely submerged in water.

- In which direction is the ball deflected in the rotating tank?
- Determine the angular velocity of the tank that is required for the thread to make angle  $\alpha$  with the vertical.

**10-9° Lazy throw** (5p) An object is thrown from a level surface with a fixed speed. The locus of points of the vertical plane that this object can reach is bounded by a parabola. Free fall acceleration is  $g$ .

- (a) Derive the equation of this parabola, if the object is thrown from the origin with the speed  $v_0$ . Consider the case when the initial velocity is vertical, and the case which maximizes the range.
- (b) An object is thrown from the centre of the base of the cone. The side surface of the cone makes an angle  $\alpha = 30^\circ$  with the floor. The minimum initial velocity that is required for the object to reach the vertex of the cone is  $v$ . Determine the minimum initial velocity  $v_1$  that is required for the object to reach the side of the cone.



**10-10° Multiple choice questions** (3p) For each question there is one correct answer. Justify your choice. Answers without justification will receive zero marks.

**(1)** Gravity on Mercury is  $K$  times smaller than on Earth. A ball dropped on Earth from height  $h$  takes time  $t$  to reach the ground. From what height should it be dropped on Mercury so that it takes the same time  $t$  to reach the surface? Neglect air resistance.

- (a)  $h/K^2$   
 (b)  $h/K$   
 (c)  $h$   
 (d)  $hK$   
 (e)  $hK^2$

**(2)** A block of brass (an alloy of density  $8730 \frac{\text{kg}}{\text{m}^3}$ ) has sides of length 3 cm, 4 cm and 5 cm. What is the maximum pressure that can be exerted by this block when it is placed on a table on one of its faces?

- (a) 4,28 kPa  
 (b) 428 kPa  
 (c) 3,43 kPa  
 (d) 257 kPa

**(3)** An astronaut in the International Space Station experiences weightlessness because...

- (a) she is outside the Earth's gravitational field;  
 (b) the attraction force of the Moon cancels out that of the Earth;  
 (c) the attraction force of the Sun cancels out that of the Earth;  
 (d) she is accelerating at the same rate as the space station.

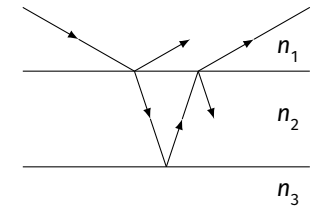
**(4)** A lift is moving upwards at a constant speed. Ignoring any friction, which statement is correct?

- (a) The kinetic energy of the lift is constant.  
 (b) The gravitational potential energy of the lift with respect to the ground is constant.  
 (c) The mechanical energy of the lift with respect to the ground is constant.  
 (d) Statements (a) and (c) are both correct, but (b) is not correct.  
 (e) Statements (a), (b) and (c) are all correct.

**(5)** A drop slide in a fairground has a steep initial slope which gradually curves into a more gentle slope. If a child drops down the slide, what happens to his speed  $v$  and the magnitude of his acceleration  $a$ , ignoring any friction forces?

- (a)  $v$  and  $a$  both increase.  
 (b)  $v$  increases,  $a$  does not change.  
 (c)  $v$  increases,  $a$  decreases.  
 (d)  $v$  decreases,  $a$  increases.

**(6)** Light rays propagate through three media of refractive indices  $n_1$ ,  $n_2$  and  $n_3$ , respectively, as shown in the diagram. Which of the following statements is correct?



- (a)  $n_1 < n_2 < n_3$   
 (b)  $n_2 < n_1 < n_3$   
 (c)  $n_1 < n_3 < n_2$   
 (d)  $n_3 < n_1 < n_2$   
 (e)  $n_3 < n_2 < n_1$