Anisotropic friction (10 points)

Force of friction is not always isotropic. Often the magnitude and direction of a friction force depend on the direction of body motion. For example, friction anisotropy can arise in the presence of "grooves" of a certain orientation on the contact surface of bodies (it is known that the coefficient of friction of oak against oak along and across the grain is equal to 0.48 and 0.34, respectively). Friction anisotropy can lead to unusual properties of motion which are studied in this problem.

What you need to know about anisotropic friction

Suppose that a surface is made of an anisotropic material. One of the most popular models of anisotropic friction suggests that there are perpendicular axes X and Y (they are called *primary*) so that the friction force $\vec{F}$ acting on a body will depend on the direction of the body motion as

$$\begin{aligned} F_x &= -\frac{|N|}{|v|}\mu_x v_x \\ F_y &= -\frac{|N|}{|v|}\mu_y v_y \end{aligned} \tag{1}$$

where F_x and F_y are the friction force components, N is a normal reaction force acting on the body, v_x and v_y are components of the velocity vector $\vec{v}$, and μ_x and μ_y are the friction coefficients along the primary axes.

Hereinafter, it is understood that the coordinate axes on the plane coincide with the primary axes. The friction coefficients are $\mu_x = 0{,}75$ and $\mu_y = 0{,}5$ unless otherwise stated.

In parts A and B a body can be considered as point-like. The plane, on which the bodies move, is horizontal in all parts of the problem.

Provide a numerical answer wherever possible.

Part A. Motion on a horizontal surface (4.0 points)

A.1	At what angle α_1 to the X axis should the body velocity vector be for the absolute value of the power of the friction force be at maximum?	0.5pt

A.2	At what angle α_2 to the X axis should the body velocity vector be for the absolute value of the power of the friction force be 1.2 times less than maximum?	0.5pt

A.3	Let the initial velocity have components $v_{0x} = 1\,\mathrm{m/s}$ and $v_{0y} = 1\,\mathrm{m/s}$. After some time the velocity component along the Y axis equals $v_{1y} = 0{,}25\,\mathrm{m/s}$. What is the velocity magnitude at this moment?	1.0pt

A.4	Let the velocity be $v_2 = 1.0\,\mathrm{m/s}$. At what angle α_3 to the X axis should the velocity vector be for the radius of curvature of the trajectory be minimum? What is this radius equal to? The free fall acceleration is $g = 9{,}8\,\mathrm{m/s^2}$.	1.0pt

A.5 In a single diagram on the XY plane, sketch the trajectories of the body launched at the angles $\alpha_4 = \pi/6$ and $\alpha_5 = \pi/3$ for the friction coefficients specified above. The magnitudes of initial velocities are the same. Solve the same problem for the friction coefficients $\mu_x = 0{,}4$ and $\mu_y = 0{,}7$. **1.0pt**

Part B. Conditions for the beginning of body movement (2.0 points)

B.1 A body of mass m is at rest at the origin. A force has been applied to it at an angle α to the X axis. The force magnitude $F(t) = \gamma t$ linearly grows with time. Find the dependence of the moment the body starts moving on α. Ignore the stagnation phenomenon. **2.0pt**

Part C. Circular motion (4.0 points)

Two identical point-like masses m are connected with a weightless inextensible rod of length $L = 1\ \mathrm{m}$, the system lies on a surface with anisotropic friction. The rod is aligned with the Y axis and does not touch the surface. One of the masses is given an initial velocity perpendicular to the rod.

C.1 For a given initial velocity v_0 find the dependence of its velocity v on the angle of rotation of the rod φ assuming that the other body remains at rest. **1.5pt**

C.2 Find the maximum value of the initial velocity $v_{0\text{max}}$ at which the other body will remain at rest. **1.5pt**

C.3 What distance will the body travel until it stops completely if the initial velocity is $v_{0\text{max}}$? **1.0pt**