The 43rd International Physics Olympiad — Theoretical Competition

Tartu, Estonia — Tuesday, July 17th 2012

- The examination lasts for 5 hours. There are 3 problems worth in total 30 points. **Please note that the points allocated to each of the three theoretical problems differ.**

- **You must not open the envelope with the problems before the sound signalling the beginning of the competition (three short signals).**

- **You are not allowed to leave your working place without permission.** If you need any assistance (broken calculator, need to visit a restroom, etc), please raise the appropriate flag ("HELP" or "TOILET") on a long handle at your seat above your seat box walls and keep it raised until an organizer arrives.

- **Your answers must be expressed in terms of those quantities which are highlighted** in the problem text, and may also contain fundamental constants if needed. So, if it is written that "the box height is a and the width is b" then a can be used in the answer, and b may not be used (unless it is highlighted somewhere else, see below). Those quantities which are highlighted in the text of a subquestion can be used only in the answer to that subquestion; the quantities which are highlighted in the introductory text of the Problem (or a Part of a Problem), i.e. outside the scope of any subquestion, can be used for all the answers of that Problem (or of that Problem Part).

- Use only the front side of the sheets of paper.

- For each problem, there are **dedicated Solution Sheets** (see header for the number and pictogramme). Write your solutions onto the appropriate Solution Sheets. For each Problem, the Solution Sheets are numbered; use the sheets according to the enumeration. **Always mark on which Problem Part and Question you are working.** Copy the final answers into the appropriate boxes of the **Answer Sheets**. There are also **Draft** papers; use these for writing things which you don't want to be graded. If you have written something that you don't want to be graded on the Solution Sheets (such as an initial and incorrect solution), cross it out.

- If you need more paper for a certain problem, please raise the "HELP" flag and tell an organizer the problem number; you will be given two Solution sheets (you may do this more than once).

- **You should use as little text as possible:** try to explain your solution mainly with equations, numbers, symbols and diagrams.

- The (first) single sound signal tells you that there are 30 min of solving time left; the (second) double sound signal means that 5 min is left; the (third) triple sound signal marks the end of solving time. **After the third sound signal you must stop writing immediately.** Put all the papers into the envelope at your desk. **You are not allowed to take any sheet of paper out of the room.** If you have finished solving before the final sound signal, please raise your flag.

Problem T1. Focus on sketches *(13 points)*

Part A. Ballistics *(4.5 points)*

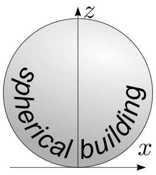
A ball thrown with an initial speed v_0 moves in a homogeneous gravitational field in the x–z plane, where the x–axis is horizontal and the z–axis is vertical. The z–axis is antiparallel to the gravitational acceleration g. Neglect the air drag.

i. (0.8 pts) Varying the angle at which a ball is thrown from the origin with a fixed initial speed v_0 allows targets within the region given by

$$z \leq z_0 - kx^2$$

to be hit. You may use this fact without proving it. Find the constants z_0 and k.

ii. (1.2 pts) Now, the ball may be launched from any point on the ground ($z = 0$), and the launching angle can be adjusted as needed. The aim is to hit the highest point of a spherical building of radius R (see the figure) with the smallest possible initial speed v_0. The ball may not bounce off the building before hitting the target. Sketch qualitatively the shape of the optimal trajectory of the ball (in the designated box on the answer sheet). Note: the points are awarded for the sketch only.



iii. (2.5 pts) What is the minimal launching speed $v_{\min}$ needed to hit the highest point of a spherical building of radius R?

La Géode, Parc de la Villette, Paris. Photo: katchooo/flickr.com

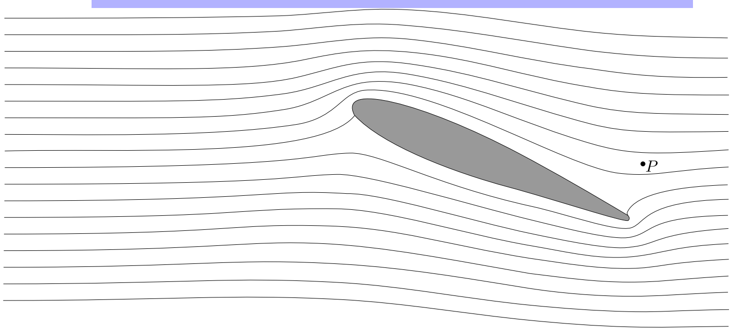
Part B. Air flow around a wing *(4 points)*

For this Problem Part, the following information may be useful. For a flow of liquid or gas in a tube,

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

holds along a streamline, assuming that the velocity v is much smaller than the speed of sound. Here ρ is the density, h is the height, g is the gravitational acceleration, and p is the hydrostatic pressure. Streamlines are defined as the trajectories of fluid particles (assuming that the flow pattern is stationary). Note that the term $\frac{1}{2}\rho v^2$ is called the dynamic pressure.

In the figure below, a cross-section of an aircraft wing is depicted, together with streamlines of the air flow around the wing as seen in the wing's reference frame. Assume that (a) the air flow is purely two-dimensional (i.e. that the velocity vectors of air particles lie in the figure plane); (b) the streamline pattern is independent of the aircraft speed; (c) there is no wind; (d) the dynamic pressure is much smaller than the atmospheric pressure $p_0 = 1.0 \times 10^5\,\mathrm{Pa}$. You may use a ruler to take measurements from the figure on the answer sheet.

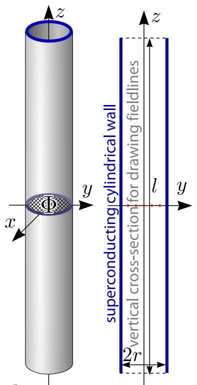


i. (0.8 pts) If the aircraft's ground speed is $v_0 = 100\,\mathrm{m/s}$, what is the speed v_P of the air at the point P (marked in the figure) with respect to the ground?

ii. (1.2 pts) In the case of high relative humidity, once the ground speed of the aircraft increases above a critical value v_{crit}, a stream of water droplets is observed trailing the wing. The droplets form at a certain point Q. Mark the point Q in the figure on the answer sheet. Explain qualitatively (using formulae and as little text as possible) how you determined its position.

iii. (2.0 pts) Estimate the critical speed v_{crit} using the following data: the relative humidity of the air is $r = 90\%$, the specific heat of air at constant pressure is $c_p = 1.00 \times 10^3\,\mathrm{J/kg \cdot K}$, the pressure of saturated water vapour is $p_{sa} = 2.31\,\mathrm{kPa}$ at the temperature of the unperturbed air $T_a = 293\,\mathrm{K}$ and $p_{sb} = 2.46\,\mathrm{kPa}$ at $T_b = 294\,\mathrm{K}$. Depending on your approximations you may also need the specific heat of air at constant volume $c_V = 0.717 \times 10^3\,\mathrm{J/kg \cdot K}$. Note that the relative humidity is defined as the ratio of the vapor pressure to the saturated vapor pressure at the given temperature. The saturated vapor pressure is defined as the vapor pressure at which vapor is in equilibrium with the liquid.

Part C. Magnetic straws *(4.5 points)*



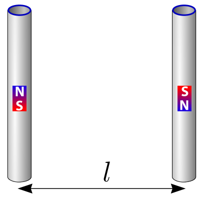
Consider a cylindrical tube made of a superconducting material. The length of the tube is l and the inner radius is r, with $l \gg r$. The centre of the tube coincides with the origin, and its axis coincides with the z-axis. There is a magnetic flux Φ through the central cross-section of the tube, i.e. at $z = 0$ for $x^2 + y^2 < r^2$.

A superconductor is a material which expels any magnetic field (the field is zero inside it).

i. (0.8 pts) In the designated box on the answer sheet, sketch five magnetic field lines which pass through the five red dots marked on the axial cross-section of the tube.

ii. (1.2 pts) Find the z-directional tension force T in the middle of the tube (i.e. the force which the two halves of the tube, $z > 0$ and $z < 0$, exert on each other).

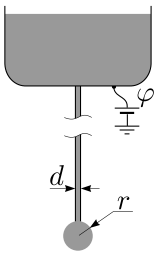
iii. (2.5 pts) Now there is another tube, identical and parallel to the first one. The second tube has an oppositely directed magnetic field to the first tube, and its centre is placed at $y = l$, $x = z = 0$ (so that the tubes form opposite sides of a square). Determine the magnetic force F which the two tubes exert on each other.



Problem T2. Kelvin water dropper *(8 points)*

The following facts about the surface tension may be useful for this problem. For the molecules of a liquid, positions at the liquid-air interface are less favourable than positions in the bulk of the liquid. Therefore, this interface has the so-called surface energy $U = \sigma S$, where S is the surface area of the interface and σ is the surface tension coefficient of the liquid. Furthermore, two fragments of the liquid surface pull on each other with a force $F = \sigma l$, where l is the length of the straight line separating the fragments.

A long metallic pipe with internal diameter d is pointing directly downwards and water is slowly dripping from a nozzle at its bottom (see the figure). Water is to be considered to be electrically conducting, its surface tension coefficient is σ and its density is ρ. Always assume that $d \ll r$. Here, r is the radius of the droplet hanging below the nozzle, which grows slowly with time until the droplet separates from the nozzle due to the gravitational acceleration g.



Part A. Single pipe *(4 points)*

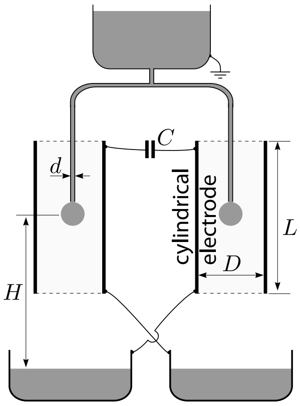
i. (1.2 pts) Find the radius $r_{\max}$ of a droplet just before it separates from the nozzle.

ii. (1.2 pts) Relative to the distant surroundings, the pipe's electrostatic potential is φ. Find the charge Q of a droplet when its radius is r.

iii. (1.6 pts) For this question, assume that r is kept constant and φ is slowly increased. The droplet becomes unstable and breaks into two pieces if the hydrostatic pressure inside the droplet becomes smaller than the atmospheric pressure. Find the critical potential $\varphi_{\max}$ at which this will happen.

Part B. Two pipes *(4 points)*

An apparatus called a "Kelvin water dropper" consists of two pipes (each identical to the one described in Part A) connected by a T-junction (see the figure). One end of each pipe is at the centre of a cylindrical electrode of height L and diameter D, where $L \gg D \gg r$. Each tube drips at a rate of n droplets per unit time. Droplets fall from a height H into conductive bowls underneath the nozzles, which are cross-connected to the electrodes as shown in the figure. The electrodes are connected via a capacitance C. There is no net charge on the system of bowls and electrodes. Note that the water container is grounded. The first droplet to fall will have a microscopic charge, which will cause an imbalance between the two sides and a small charge separation across the capacitor.



i. (1.2 pts) Express the magnitude of the charge Q_0 of the droplets which fall at the instant when the capacitor's charge is q, in terms of $r_{\max}$ (as found in Part A-i). Neglect the effect described in Part A-iii.

ii. (1.5 pts) Find the dependence of q on time t by approximating it as a continuous function $q(t)$ and assuming that $q(0) = q_0$.

iii. (1.3 pts) The dropper's functioning can be hindered by the effect considered in Part A-iii. Additionally, there is a limit $U_{\max}$ to the maximum voltage between the electrodes, determined by the electrostatic push between a droplet and the bowl beneath it. Find $U_{\max}$.

Problem T3. Protostar formation *(9 points)*

Let us model the formation of a star as follows. A spherical cloud of sparse interstellar gas, initially at rest, starts to collapse due to its own gravity. The initial radius of the cloud is r_0 and its mass is m. The temperature of the surroundings (which are much sparser than the gas) and the initial temperature of the gas are uniformly T_0. You may assume that the gas is ideal. The average molar mass of the gas is μ and its adiabatic index is $\gamma > \frac{4}{3}$. Assume that $G\frac{m\mu}{r_0} \gg RT_0$, where R is the gas constant and G is the universal gravitational constant.

i. (0.8 pts) During much of the collapse, the gas is so transparent that any heat generated is immediately radiated away, i.e. the cloud stays in thermodynamic equilibrium with its surroundings. By what factor n does the pressure increase when the radius is halved ($r_1 = 0.5r_0$)? Assume that the gas density stays uniform.

ii. (1 pt) Estimate the time t_2 taken for the radius to shrink from r_0 to $r_2 = 0.95r_0$. Neglect the change of the gravitational field at the position of a falling gas particle.

iii. (2.5 pts) Assuming that the pressure stays negligible, find the time $t_{r\to 0}$ taken for the cloud to collapse from radius r_0 down to a much smaller radius, using Kepler's Laws for elliptical orbits.

iv. (1.7 pts) At some radius $r_3 \ll r_0$, the gas becomes dense enough to be opaque to the heat radiation. Calculate the amount of heat Q radiated away during the collapse from radius r_0 down to radius r_3.

v. (1 pt) For radii smaller than r_3 you may neglect heat radiation. Determine how the temperature T of the cloud depends on its radius $r < r_3$.

vi. (2 pts) Eventually we cannot neglect the effect of the pressure on the dynamics of the gas and the collapse stops at $r = r_4$ (with $r_4 \ll r_3$). However, the radiation can still be neglected and the temperature is not yet high enough to ignite nuclear fusion. The pressure of such a protostar is no longer uniform, but rough estimates with inaccurate numerical prefactors can still be made. *Estimate* the final radius r_4 and the corresponding temperature T_4.