

**1 Snow on benches** (3 points)

This winter inhabitants of Alūksne noticed regularly distributed heaps of snow on the benches around the local open-air stage (see the photo). Local authorities also shared photos showing how these heaps were changing in time. Propose and justify a mechanism that would describe formation of such heaps of snow.

**2 Quadcopter** (2 points)

At the start of a bicycle race, the reporter started off a quadcopter to follow contestants of the ride at close distance. Assuming the bicyclists are riding on a straight highway with a constant speed  $v = 30$  km/h, calculate how far from the start the quadcopter will be able to continue filming the race if the reporter

- stays back at the start;
- drives a car to the point on the highway  $L = 20$  km away from the start and waits there.

The battery of the quadcopter holds enough charge to last for  $t_0 = 30$  min of flight independent of the speed. The reporter knows the distance to the quadcopter at all times. The quadcopter returns to the reporter with the speed  $u = 50$  km/h.

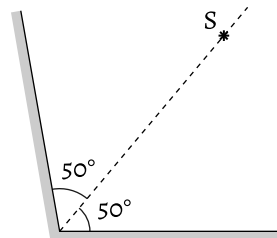
**3 A rock in ice** (3 points)

A rock of mass 1 kg is frozen into an ice floe of mass 20 kg drifting in an ocean. The floe receives 10 J of energy every second from various sources. After what period of time will the floe start sinking?

Latent heat of fusion and density of ice are 244 kJ/kg and 0.925 g/cm<sup>3</sup>, respectively. Densities of seawater and rock are 1.024 g/cm<sup>3</sup> and 2.5 g/cm<sup>3</sup>, respectively.

**4 Images in mirrors** (4 points)

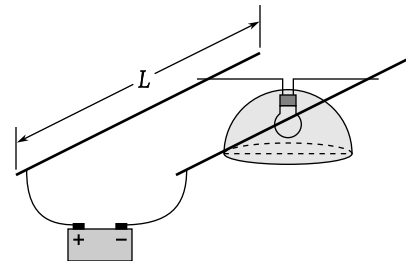
Two plane mirrors form an angle  $\alpha = 100^\circ$ . A point light source  $S$  is located on the bisector of this angle. How many images of the source  $S$  will such an optical system produce? How many of these can one see looking from a large distance to mirrors?

**5 Water supply** (4 points)

In the house, original hot water supply system is deployed. It consists of a large reservoir from which hot water is distributed to the rooms and a device that automatically maintains water level in the reservoir by pouring in boiling water. At usual consumption rate, the temperature of water in the reservoir is  $T_1 = 60^\circ\text{C}$ . What will the new stationary water temperature be if consumption rate is doubled? The temperature of the environment is  $T_0 = 20^\circ\text{C}$ . The rate of heat loss from the reservoir to the environment is proportional to the difference between the temperature of water in the reservoir and the temperature of the environment.

**6 Resistance of the wire** (3 points)

Two mutually isolated parallel wires span the length of a  $L = 20$  m long hallway. The wires are connected to the  $U_0 = 12$  V voltage source, which is located at one end of the hallway. An incandescent lamp of nominal power  $P_0 = 60$  W designed for the voltage of 12 V can slide along the wires. One can note that when the lamp is at the further (with respect to the source) end of the hallway, its light becomes dimmer, as the voltage across it is  $U = 11$  V. Determine the resistance of each metre of the wire and the power of the lamp at the further end of the hallway. Assume that the resistance of an incandescent lamp is proportional to the square root of the voltage across it,  $R \sim \sqrt{U}$ .

**7 Crate on a wedge** (4 points)

A crate is sliding down the light wedge of incline angle  $\alpha$ , which is located on the horizontal floor. Friction factor between the wedge and floor is  $\mu_0$ . What minimal value can the friction factor  $\mu$  between the wedge and the crate have in order for the wedge to remain at rest? Neglect the mass of the wedge.

**8 Tesla** (3 points)

A battery of the electric car can store 50 kWh of energy, and when fully charged will let the car cover 500 km at 100 km/h.

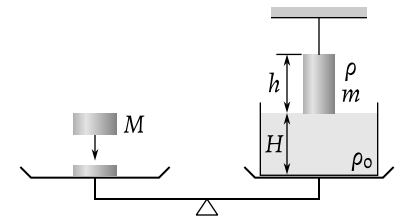
(a) Assuming the efficiency of the car to be 80% – that includes electric-to-mechanical energy transformation as well as losses in transmission – determine the power of the electric motor.

(b) What volume of gasoline would have been consumed by an identical car but with an internal combustion engine if its efficiency would have been 25%? Energy density of gasoline is 34 MJ/l.

**9 Balanced scale** (3 points)

A flask with water on a scale is balanced by a counterweight. A metal cylinder of mass  $m$ , height  $h$  and density  $\rho$  is hanging on a string in such a way that it barely touches the surface of water. The cylinder is then immersed in water by half of its height. What is the mass  $M$  of an additional counterweight that one needs to add to the left side of the scale in order to keep the scale balanced?

The height of the cylinder is less than the height of water column in the flask, i.e.  $h < H$ . Water density is  $\rho_0$ . Neglect the vertical displacement of the platforms of the scale.



**Multiple choice questions** (5 points)  
For each question there is one correct answer. Justify your choice. Answers without justification will receive zero marks.

**o-1** Two bodies are moving on a straight line. The positions of the bodies at successive 0.2-second time intervals are represented in the table below.

$t, s$	$x_1, m$	$x_2, m$
0.0	0.2	0.0
0.2	0.4	0.4
0.4	0.7	0.8
0.6	1.1	1.2
0.8	1.6	1.6
1.0	2.2	2.0
1.2	2.9	2.4
1.4	3.7	2.8

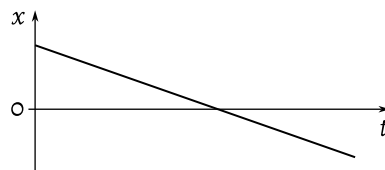
Do the bodies ever have the same speed?

- (A)** No.
- (B)** Yes, at  $t = 0.2$  s and  $t = 0.8$  s.
- (C)** Yes, at  $t = 0.2$  s.
- (D)** Yes, at  $t = 0.8$  s.
- (E)** Yes, at some time between  $t = 0.4$  s and  $t = 0.8$  s.

**o-2** A hollow toy boat is floating in a bath. If you take a teaspoon full of water out of the bath and put it in the boat, what happens to the water level in the bath?

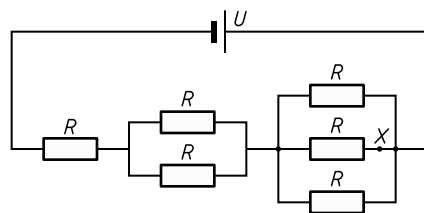
- (A)** The level goes down.
- (B)** The level goes up.
- (C)** The level stays the same.
- (D)** There is not enough information to say.

**o-3** The following represents a position-time graph ( $x-t$ -diagram) for an object. The  $x$  axis is pointing to the right. Which of the following describes the motion best?



- (A)** The object always moves to the right.
- (B)** The object always moves to the left.
- (C)** The object moves to the right at first. Then it moves to the left.
- (D)** The object moves down an inclined plane.

**o-4** What is the total resistance of the circuit?



- (A)**  $11R/6$
- (B)**  $3R$
- (C)**  $6R/11$
- (D)**  $R/3$

**o-5** What is the current at the point X (see the diagram from the previous question)?

- (A)**  $2U/(11R)$
- (B)**  $6U/(11R)$
- (C)**  $U/(6R)$
- (D)**  $3U/(11R)$

**o-6** When a metal bar is cooled it contracts. Which of the following is true?

- (A)** The mass remains constant, the density decreases.
- (B)** The density and mass increase.
- (C)** The mass remains constant, the density increases.
- (D)** The density and mass are unchanged.

**o-7** A hot air balloon is descending at a steady speed of 11 m/s. The pilot drops a sandbag, which takes 7 s to fall to the ground. What was the height of the balloon when the sandbag was released?

- (A)** 168 m
- (B)** 245 m
- (C)** 322 m
- (D)** 528 m

**o-8** A cube of metal has sides of length  $x$ . The electrical resistance between opposite faces of the cube is ...

- (A)** directly proportional to  $x$ .
- (B)** inversely proportional to  $x$ .
- (C)** directly proportional to  $x^2$ .
- (D)** independent of  $x$ .

**o-9** A 3.6 V mobile phone battery can produce 0.70 A of current for an hour. This can be charged using a square solar panel 25 cm on each side. What time is needed to charge the battery assuming an efficiency of 10% and an incident solar power of 1.0 kW/m<sup>2</sup>?

- (A)** 0,10 h
- (B)** 0,28 h
- (C)** 0,40 h
- (D)** 1,5 h

**o-10** A light dependent resistor is connected to an ideal 12 V source and placed in the open on a horizontal surface in the middle of a desert. When is the power dissipated in the resistor the highest?

- (A)** At dawn.
- (B)** At mid-morning.
- (C)** At noon.
- (D)** At midnight.

**1 Snow on benches** (3 points)

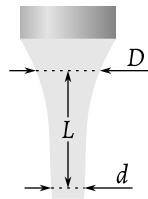
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**2 Helicopter** (3 points)

Estimate the power of the engine that is required to hover a helicopter of mass  $m = 500$  kg if the length of its propeller blades is  $L = 3$  m. Assume that below the blades, air is moving down in a uniform flow.

**3 Tap water** (4 points)

A vertical water stream is flowing from the tap. The diameter of the stream decreases from  $D = 3$  mm to  $d = 2$  mm when flowing through  $L = 3$  cm height difference. Determine the volume of water that flows out of the tap per second.

**4 Achromatic lens** (5 points)

Refractive index of glass, and thus also the optical power of lenses, depends on the wavelength of light. Due to that *chromatic aberration* can be observed. This phenomenon makes it hard to obtain sharp images of coloured objects in optical systems. In order to partly compensate for this drawback, lenses can be made up of two tightly sealed glasses of different types.

Calculate optical powers  $D_K$  and  $D_F$  of lenses made of two types of glass – crown glass BK7 and flint glass F2 – if, when combined, they should make up a lens of the optical power  $D_{KF} = 10 \text{ m}^{-1}$  for both red and blue light. Calculate the optical powers  $D_K$  and  $D_F$  of both lens components for green light ( $\lambda = 550 \text{ nm}$ ). Optical power of a lens of the given shape in air is proportional to  $(n - 1)$ , where  $n$  is the refractive index of the glass.

$\lambda, \text{nm}$	$n_K$	$n_F$
480	1.5228	1.6331
550	1.5185	1.6237
700	1.5130	1.6126

**5 Two coils** (4 points)

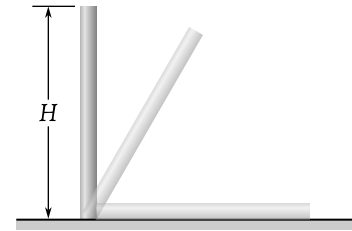
Two identical wire coils are located close to each other. One of them is attached to the sine-wave AC voltage source in series with an ammeter. The leads of the other coil are connected to another ammeter. Readings of ammeters are 1,0 A and 0,2 A. When the circuit of the second coil is broken, what will the reading of the remaining ammeter be? Neglect the active resistance of coils. Assume wires, ammeters and the source to be ideal.

**6 Falling pole** (4 points)

A vertical telegraph pole of height  $H = 6$  m starts falling on a level surface without slipping.

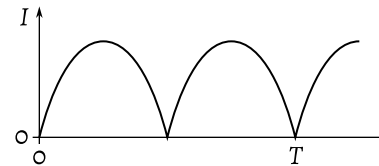
(a) Determine the speed of the free end of the pole when it touches the surface.

(b) Which of the points of the pole will at all times have the same speed as an object freely falling from the same initial height?

**7 Charging the battery** (4 points)

A rechargeable battery of 40 A·h capacity is charged with rectified AC current (see fig.). An ammeter in the circuit shows the current of 3 A. What time will it take for initially empty battery to fully charge up? Assume that all the current is used for charging.

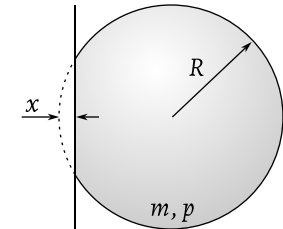
Capacity of a battery is the charge that this battery can release during discharge. An ammeter measures the RMS value of current.

**8 Holding the atmosphere** (3 points)

Estimate the minimal radius of a planet that would suffice to hold carbon dioxide atmosphere for a long period of time. Justify your solution. Average density of the planet (not including the atmosphere) is  $3.5 \text{ g/cm}^3$ , its surface temperature is 250 K.

**9 Rebound in detail** (4 points)

During a weak collision of a basketball with a wall, it deforms as shown in the figure, the deformation  $x$  of the ball is much less than its radius  $R$  and it can be assumed that air pressure  $p$  inside the ball remains constant all the time. Estimate the duration of collision between the ball and the wall neglecting the elasticity of the shell of the ball. Also, give a numerical estimate if the mass of the ball  $m = 0.5$  kg, its radius  $R = 15$  cm, air pressure inside of it  $p = 200$  kPa.

**10 Snow sparkles** (4 points)

Fresh snow sparkles in the light of winter sun. Estimate the average distance between neighbouring sparkles assuming that snow surface is covered by randomly oriented flat crystals 1 mm in size. Angular diameter of the Sun is  $0.5^\circ$ .